Higher-Dimensional Space and Regular Polytopes: THE EXPLORATION OF SHAPES

NAME: Jose Miguel Valdes Rodriguez
SCHOOL: Ecole John Stubbs Memorial
GRADE: 5

Introduction

My name is Jose Miguel Valdes Rodriguez. I am a 10 years old boy passionate about Mathematics, Chemistry, Astronomy, Biology, and all sciences. I would like to participate on the Vancouver Island Science Fair, to discuss one of the topics that fascinates me: higher dimensional space.

My exhibit should get you familiar with understanding that topology isn't limited to 3-dimensional space. Since we perceive our world in three dimensions, we cannot perceive the fourth and higher dimensions. But we can still do mathematical analysis using analogies even if we can't comprehend those extra dimensions. You should know that we can do the analysis of 4 dimensional objects using that process of analogy. By analogy I mean understanding how a 2D being would experience the third dimension to understand how we would understand the fourth. It should let you understand the shapes that exist in 4-dimensional space and up. I will also review the polyhedra, 2D figures, and the details of 1 and 0 dimensions.

I will talk about wireframe projection; a transparent projection of the object. Usually when you look at an object, you can only see the faces that are in the front of you. But the wireframe projection I'll use on the models I will be displaying, lets you see through the objects better than other projections like "shadow".

There are two types of wireframe projections: the oblique and the perspective. In the oblique, the two cubes you see are the same size. All you see is the two cubes and then connection lines connecting the two cubes. Alternatively, there is the perspective projection. Here you look straight at one of the eight cubes that make it up. You see a cube inside another cube, and the two cubes are also connected. The perspective projection is more preferred than oblique projection, although I won't use it in my models of a hypercube.

There are images of hypercubes in Wikipedia that represent their oblique projections accurately up to 15 dimensions. However, there are also projections of the simplex, the simplest regular polytope in any dimension, up to 20 dimensions. And there are projections of the cross-polytope (dual to the hypercube of the same dimension) up to 10 dimensions. Though these are more "top-viewed" than oblique.

Because there are many very complicated shapes to model, I won't be able to build models for all the possible shapes. Instead, I will put together a slideshow to illustrate the more complex objects. I will also talk about non-polytopes too. There is a huge diversity of shapes that explodes the amount of shapes in a dimension to infinity. This starts at 2 dimensions. I think the diversity of shapes is an important topic for discussion because there is a vast amount of space, we can't comprehend due to our perception of our world. Understanding what we can't imagine is a great way to test our abilities to trying to see what our eyes are incapable of seeing.

Procedure

On my exhibit, I will have homemade models of the objects made of wood sticks and hot glue. I will have a few of the 2D polygons, 3D polyhedra, 4D polychora, and 5D and up polytopes. The models will be wireframes projections, and they will have two types; the oblique and the perspective. The only oblique projections in 4 and higher dimensions that I will show will be the hypercube (equivalents of the cube and tesseract). The rest of the models that I will show will be perspective projection since there is rarely any image of an oblique 4 and higher dimensional shape other than a hypercube of that dimension.

I will explain to the visitors of my stand the analogies we use to imagine what a 4D world would look like. I will walk them through the realms of the fourth and higher dimensions. I will explain how to build the models, how to imagine the object, and the characteristics of the object. I'll take the example of the tesseract, which is the 4D equivalent of the cube. By analogy, extending a cube in the fourth dimension will give us the tesseract. This analogy comes from the fact that the cube is the extension of the square in the third dimension.

I will also explain the formulas to find the components of the object, and other properties like volume, surface area, and more. I will prove those formulas to the visitors so that they can understand how mathematics relates to dimensions. For example, there are 2^n corners in an N-dimensional hypercube. Thus, there is 1 corner in a point, 2 corners in a line, 4 corners in a square, 8 corners in a cube, and thus there are 16 corners in a tesseract, 32 corners in a penteract (a penteract is the 5D generalization of the tesseract), and the pattern continues.

There are also 2*n bounding n-1 dimensional hypercubes in an n-dimensional hypercube. So, there are 2 bounding points in a line, 4 bounding lines in a square, 6 bounding squares in a cube, and so there are 8 bounding cubes in a tesseract, 10 bounding tesseracts in a penteract, 12 bounding penteracts in a 6D hypercube, and so on.

I will also explain what a dimension is, why we can't comprehend the higher dimensions, and how to map higher dimensional axes on a piece of paper. I will explain how I designed the models, and I will have some reference books available if the judges or visitors are interested. I will have a backboard with cool shapes in higher dimensions and I will display a slide show of higher dimensional objects which I will be happy to discuss them as well. I will also ask the visitors to guess which dimension they are looking at.

Results

The models would represent what an object in a higher dimension looks like so that you can understand the context of the object. Note that what is in fact bizarre is that we can model the four independent dimensions in 3D space, but we cannot comprehend 4D space. You will notice that the components of the object are somewhat distorted due to the projection so we can never get a perfect image of a 4-dimensional object. I will also model 5 and 6-dimensional objects just for you to see that we can model the objects down into 3D space, even though we can't comprehend these dimensions of space. And you might ask a question: don't we live in a 4D space if we consider the dimension of time? Well, we won't consider time as a dimension of space because you can throw a ball through spatial dimensions, but you cannot throw a ball to next year.

Conclusions

In summary, the information I will try to convey is that even though we can't see something, we can use the analogy in our world to imagine the realms of a world hidden from us, due to our perception. I know that we can't see the object, but maybe, we humans can change the way we see and give ourselves a push to four-dimensionality. It has always fascinated me that our world is not composed of that many dimensions, but our imagination is limitless. It will continue to fascinate me, and it might fascinate you too that we can theoretically imagine… the unimaginable.

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